Dispersion Imaging Scheme

Here, the dispersion analysis scheme as normally adopted in the MASW method is described as it appears in Ryden et al. (2004) with modified graphical illustrations More detailed description can be found in Park et al. (1998a; 2001).

An N-channel record mr_N is defined as an array of N traces collected by one of the aforementioned acquisition methods: $mr_N=r_i$ (i=1, 2, …, N) with its frequency-domain representation of $MR_N(\omega)=R_i(\omega)=FFT[r_i]$ ($i=1, 2, ..., N$). Then, $R_i(\omega)$ can be written as a product of amplitude, $A_i(\omega)$, and phase, $P_i(\omega)$, terms: $R_i(\omega)=A_i(\omega)P_i(\omega)$. $A_i(\omega)$ changes with both offset (i) and angular frequency (ω) due to spherical divergence, attenuation, and the source spectrum characteristics. $P_i(\omega)$ is the term that is determined by phase velocity (c) of each frequency:

$$P_i(\omega) = e^{-j\Phi_i(\omega)}, \tag{1}$$

where

$$\Phi_i(\omega) = \omega x_i / c \ = \omega \{x_1 + (i-1)dx\}/c \ . \tag{2}$$

Consider one specific frequency (e.g., 20 Hz) of $R_i(\omega)$. Its time-domain representation will be an array of sinusoid curves of the same angular frequency, but with different amplitude and phase. Since the amplitude does not contain any information linked to phase velocity, $R_i(\omega)$ can be normalized without loss of significant information:

$$R_{i,norm}(\omega) = R_i(\omega)/|R_i(\omega)| = P_i(\omega) . \tag{3}$$

Fig. 1a shows an array of normalized sinusoid curves for an arbitrary frequency of 20 Hz propagating at another arbitrary phase velocity of 140 m/sec. Sinusoid curves in the figure have the same phase along a slope (S_0) of the phase velocity, whereas they have different phase along the slopes of other phase velocities, as indicated in the figure. Therefore, if the curves are summed together within a finite time length (e.g., one period) along the slope S_0, then it will give another sinusoid curve of finite length whose amplitude (A_S) is N. On the other hand, A_S will be smaller than N if the summation is performed along other slopes. This principle is the key element of the dispersion analysis employed in the MASW method. In practice the summation can be performed in a scanning manner along many different slopes specified by different phase velocities changing by small increments (e.g., 5 m/s) within a given range (e.g., 10 m/s – 500 m/s). The result of each summation as represented by amplitude (A_S) of summed sinusoid curves can be then displayed in a 2-D format (i.e., phase velocity versus A_S). In this 2-D scanned curve, the phase velocity that gives the maximum amplitude ($A_{S,max}$) will be the correct value being sought (Fig. 1b). As illustrated in Fig. 1b, the 2-D scanned curve has one main lobe with a peak amplitude $A_{S,max}$ and many side lobes on both sides. It is the sharpness of this main lobe that affects the resolution and accuracy of the analyzed dispersion relationship. In Park et al. (2001) a detailed parametric examination of the scanning method on its resolution in response to change in such parameters as N, c, dx, and ω is presented. Generally the sharpness of the peak A_S increases with N, and this means that more traces will ensure higher resolution in the determination of a phase velocity. This effect is illustrated in Fig.1b for N values of 2, 20, and 80 traces. A_S has been normalized with respect to N so that the peak value is one in all three cases.

The aforementioned summation operation can actually be accomplished in the frequency domain:

$$A_s(c_T) = e^{-j\delta_{1,T}} R_{1,norm}(\omega) + e^{-j\delta_{2,T}} R_{2,norm}(\omega) + \quad + e^{-j\delta_{N,T}} R_{N,norm}(\omega)\,, \tag{4}$$

where

$$\delta_{i,T} = \omega\left[\{x_1 + (i-1)dx\}/c_T\right] \tag{5}$$

This is a phase term that increases with offset (distance) (x) and determined by a testing phase velocity (c_T) within a scanning range. $A_S(c_T)$ is a complex number whose absolute value ($|A_S(c_T)|$) is the same as the amplitude (A_S) of summed sinusoid wave in time domain previously explained.

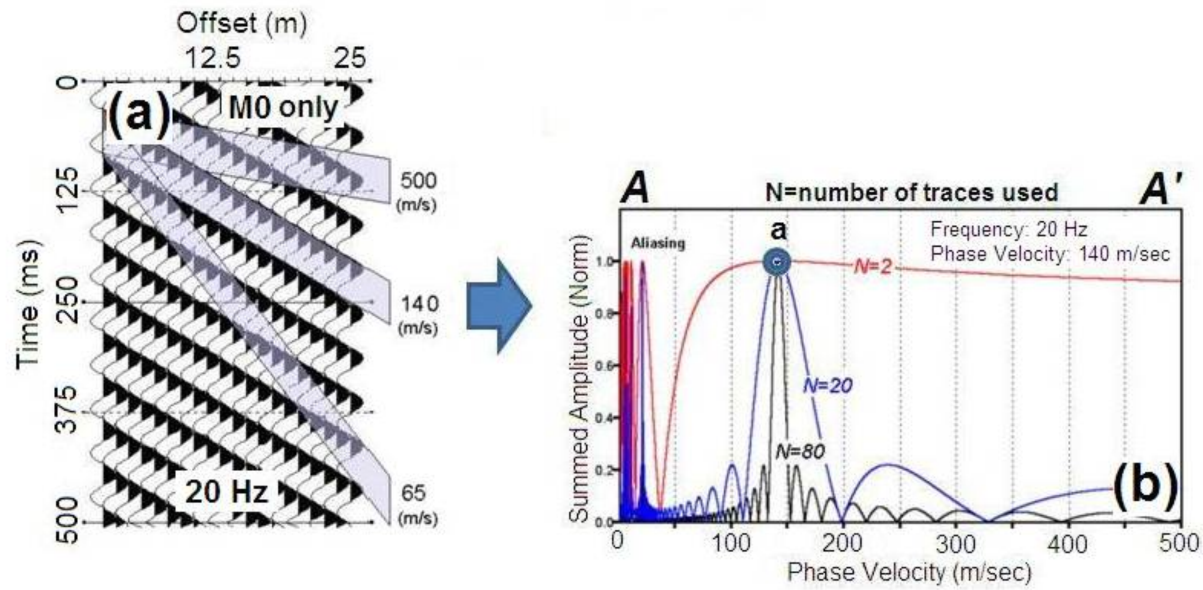

Fig. 1. (a) A surface wave of an arbitrary frequency of 20 Hz with an arbitrary phase velocity of 140 m/sec, and (b) curves of summed amplitudes for different number of traces (offset ranges).

When seismic wave propagation invokes multi-modal characteristics (like inclusion of higher modes) or includes different types of waves (like body and surface waves together), a multiple number of phase velocities can exist at the same frequency. This multi-phase-velocity case can be treated as a linear superposition of individual single-phase-velocity cases. For example, if there exists a fundamental mode (M0) (Fig. 2a) and one higher mode (M1) (Fig. 2b) at the same frequency with different phase velocities, the measured wavefield then would be the same as a superposition (Fig. 2c) of the two separate records (Fig. 2a and 2b). This means that if the phase-velocity scanning is applied to this multi-modal record, the resulting scanned curve will be the same as a superposition (Fig. 2e) of the two individual scanned curves (Fig. 2d) obtained from each single-mode record. To identify dispersion curves, all 2-D curves at different frequencies are assembled to a 3-D image showing the energy distribution as a function of phase velocity and frequency. Park et al. (2001) shows that in this case, however, the superposition involves a scaling term determined by the relative energy partitioning between the two modes. Therefore, two main lobes appear with different peak amplitudes. This is additional information that would be critical for the study of energy partitioning between different modes or different types of seismic waves along the survey line.

Fig. 3 shows modeling cases to illustrate the imaging principles for aforementioned single-and multi-modal cases.

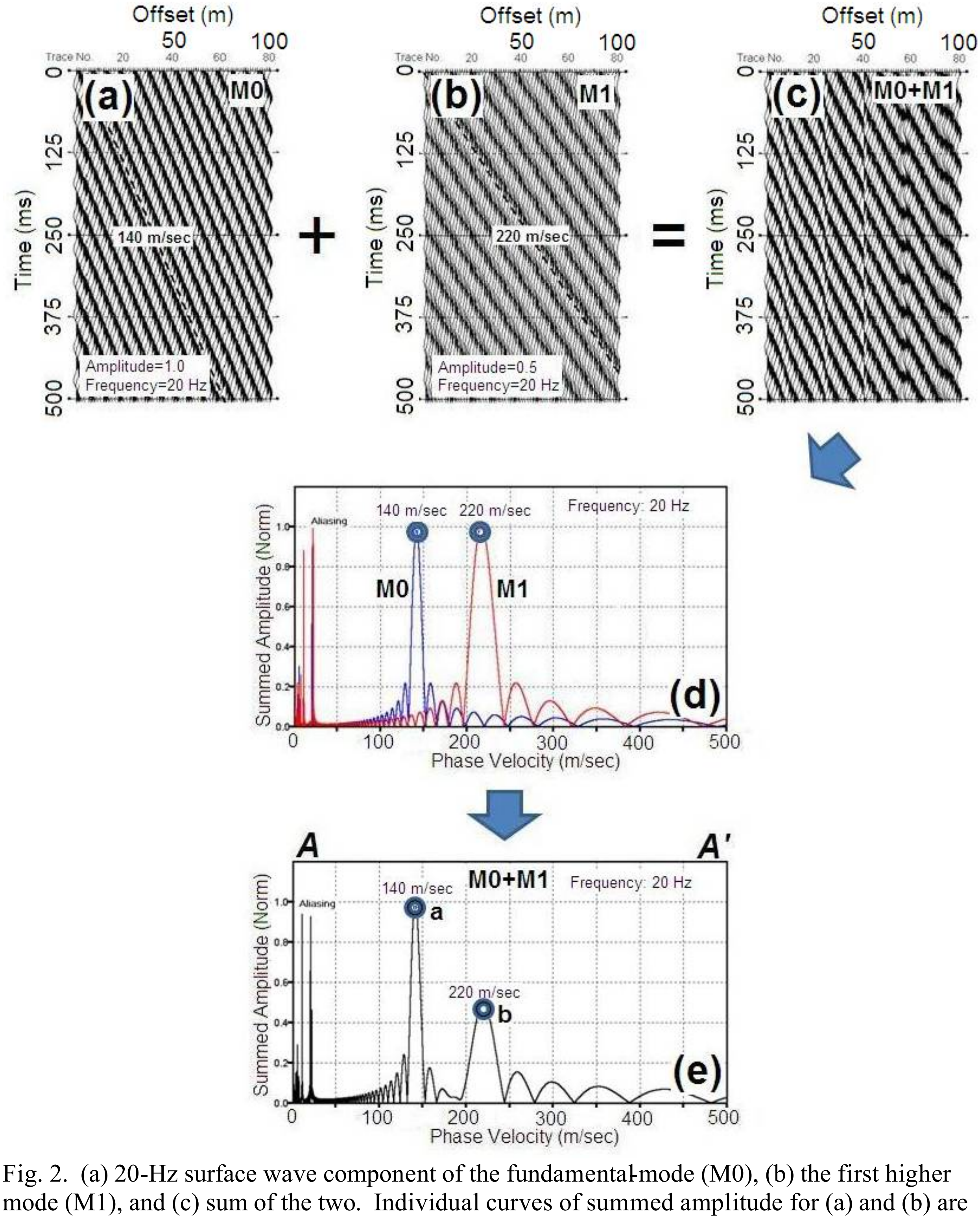

Fig. 2. (a) 20-Hz surface wave component of the fundamental-mode (M0), (b) the first higher mode (M1), and (c) sum of the two. Individual curves of summed amplitude for (a) and (b) are shown in (d), and sum of these two curves is shown in (e).

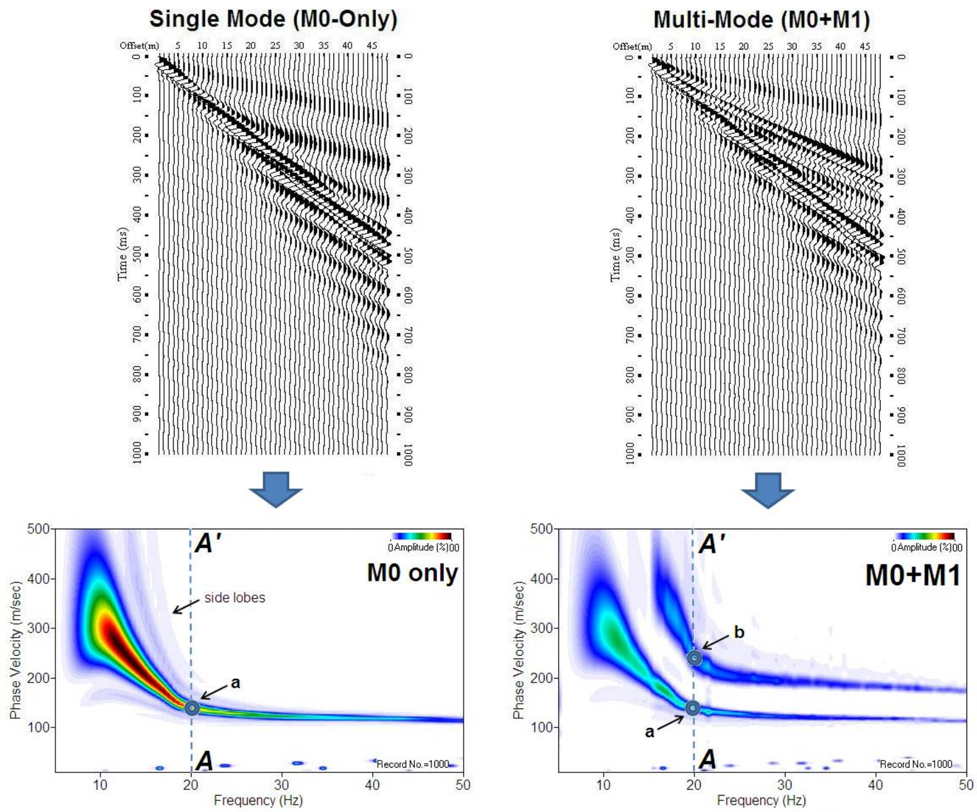

Fig. 3. Synthetic records of single- and two-mode cases (top), and their dispersion images. The line A-A’ marked on the images can be associated with the same lines indicated in Figures 1 and 2.